

# On the 1-loop calculations of softly broken fermion-torsion theory in curved space using the Stückelberg procedure

G. de Berredo-Peixoto

Departamento de Física, ICE, Universidade Federal de Juiz de Fora, Campus  
universitário  
Juiz de Fora, MG 36036-330 Brazil  
guilherme@fisica.ufjf.br

## Abstract

The soft breaking of gauge or other symmetries is the typical Quantum Field Theory phenomenon. In many cases one can apply the Stückelberg procedure, which means introducing some additional field (or fields) and restore the gauge symmetry. The original softly broken theory corresponds to a particular choice of the gauge fixing condition. In this paper we use this scheme for performing quantum calculations for fermion-torsion theory, softly broken by the torsion mass in arbitrary curved spacetime.

Keywords: Softly symmetry breaking, Renormalization, Propagating torsion, curved space.

PACS numbers: 04.62.+v, 11.10.Gh, 11.15.-q, 11.30.-j.

## 1 Introduction

Softly broken gauge symmetries are frequently an important property in Quantum Field Theory (QFT). One example is supersymmetry, which must be (most likely softly) broken in order to address the phenomenological applications and eventually experimental tests [1]. Another interesting application of the softly symmetry breaking is the effective QFT approach to the propagating torsion [2, 3, 4]. The completely antisymmetric component of torsion can be described by the dual axial vector coupled to fermions through the axial vector current. The presence of the symmetry breaking mass of the axial vector is required for the consistency of the effective theory in the low-energy sector. Indeed, the massive counterterm shows up at 1-loop level.

In many cases, one is interested not only in the classical aspects of the theory, but also in the derivation of quantum corrections. The subject of the present paper is the calculation of 1-loop effective action for the softly broken gauge theory of propagating torsion in curved space-time. Here, the kinetic term and the interactions terms in the classical action are gauge invariant while the massive terms are not. Consequently, the standard methods for evaluating the effective action face serious technical difficulties. As a strategy, we shall apply the Stückelberg procedure [5], that is, we are going to restore the gauge

symmetry by introducing an extra field or a set of fields. More details and applications of the method to models with softly broken gauge symmetry in curved spacetime can be found in Ref. [6].

We are going to show that our approach means much simpler and more efficient calculation of quantum corrections. The difference is especially explicit for the massive torsion-fermion system which was originally elaborated in Ref. [3]. The present method provides an independent verification of our previous result in Ref. [3] and also enables one to perform the calculations in an arbitrary curved space-time, something that was impossible in the framework used in Ref. [3].

## 2 Massive softly broken torsion field coupled to fermion

Torsion  $T^\alpha_{\beta\gamma}$  is an independent (along with the metric) quantity describing the spacetime manifold. It is defined by the relation (see, e.g., Refs. [4, 7] for introduction)

$$\Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta} = T^\alpha_{\beta\gamma}.$$

It proves useful to divide torsion into three irreducible components  $T_\mu$ ,  $S_\mu$ ,  $q_{\alpha\beta\mu}$  as already known in literature. The interaction with the Dirac fermion is described in a quantum consistent way by the action for the theory of effective fermion-torsion system [2] (see also Refs. [3, 4]),

$$S_{tf} = \int d^4x \sqrt{g} \left\{ -\frac{1}{4} S_{\mu\nu}^2 - \frac{1}{2} M^2 S_\mu^2 + i\bar{\psi}\gamma^\mu (\nabla_\mu + i\eta\gamma^5 S_\mu) \psi + m\bar{\psi}\psi \right\}. \quad (1)$$

Here  $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$ ,  $M$  is the torsion mass, we consider only one non-vanishing component of torsion,  $T_{\alpha\beta\gamma} = -\frac{1}{6}\varepsilon_{\alpha\beta\gamma\mu} S^\mu$ , and  $\nabla_\mu$  is the covariant derivative without torsion.

To calculate the 1-loop effective action for this model, one has to apply the generalized method of Schwinger-DeWitt [8] in the transverse vector space, as was done in Ref. [3]. Following the approach discussed in Ref. [6], one can apply the Stückelberg procedure by introducing a new scalar field,  $\varphi$ , and restoring the gauge symmetry in the following way:

$$S'_{tf} = \int d^4x \sqrt{g} \left\{ -\frac{1}{4} S_{\mu\nu}^2 + \frac{1}{2} M^2 \left( S_\mu - \frac{\partial_\mu \varphi}{M} \right)^2 + i\bar{\psi}\gamma^\mu (\nabla_\mu + i\eta_1\gamma^5 S_\mu) \psi + m\bar{\psi} \exp\left(\frac{2i\eta\gamma^5\varphi}{M}\right) \psi \right\}, \quad (2)$$

The gauge symmetry must be supplemented by  $\varphi \rightarrow \varphi' = \varphi - M\beta$ . The original theory (1) is restored when we use the gauge fixing condition  $\varphi = 0$ .

### 3 One-loop effective action and quantum (in)consistency

In order to obtain the one-loop divergences for the original theory (1), one has to put  $\varphi = 0$  in the general expression for the divergences of theory (2), which can be computed by the standard Schwinger-DeWitt method. Then the final result reduces to

$$\begin{aligned} \Gamma_{\text{div}}^{(1)} = & -\frac{\mu^{n-4}}{(4\pi)^2(n-4)} \int d^n x \sqrt{g} \left\{ 4\eta^2 m^2 S^\mu S_\mu - \frac{1}{3}\eta^2 S_{\mu\nu}^2 + 4i\eta^2 \frac{m^2}{M^2} \bar{\psi} \gamma^\mu \mathcal{D}_\mu^* \psi + \right. \\ & \left. + 2i\eta^2 \bar{\psi} \gamma^\mu \mathcal{D}_\mu \psi + \left( \frac{8\eta^2 m^3}{M^2} - 4\eta^2 m \right) \bar{\psi} \psi + \frac{2\eta^2 m}{3M^2} \bar{\psi} R \psi + \frac{8\eta^4 m^2}{M^4} (\bar{\psi} \psi)^2 \right\}, \end{aligned} \quad (3)$$

where  $\mathcal{D}_\rho = \nabla_\rho + i\eta\gamma^5 S_\rho$  and  $\mathcal{D}_\rho^* = \nabla_\rho - i\eta\gamma^5 S_\rho$ .

It is worth mentioning that the above result is more general than the result of Ref. [3]. Indeed, it is valid in curved spacetime, where a new non-minimal coupling with curvature shows up. Of course this term is relevant for dynamics of Dirac particles in the curved background, but the theory contains the  $(\bar{\psi}\psi)^2$ -term which has non-trivial consequences. In fact, at 2-loop level, this term is responsible for appearance of the Feynman diagrams drawn in Fig. 1.

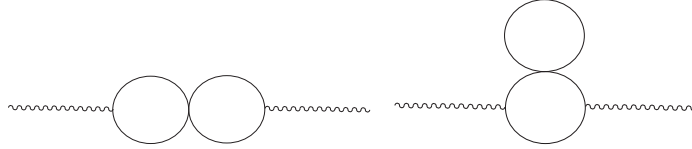


Figure 1: 2-loop diagrams with quartic fermionic vertices. The wavy lines correspond to torsion propagator, and the others to the fermion one.

Detailed calculation of these diagrams reveals the appearance of the  $(\partial_\mu S^\mu)^2$ -type counterterm, which introduces longitudinal degrees of freedom breaking unitarity. This undesirable contribution can not be compensated by another 2-loop diagrams, unless some artificial fine-tuning between different coupling constants takes place.

Even if theory (1) is not consistent at the quantum level, it can perhaps mimic some fundamental theory, as an effective theory. In this sense, it would be interesting to study the phenomenological consequences of the coupling term,  $gR\bar{\psi}\psi$ . For instance, this term seems to introduce some kind of modified fermion mass, giving rise to an interesting non-trivial effect on the mass renormalization.

### Acknowledgments

The author is grateful to Prof. I.L. Buchbinder and Prof. I.L. Shapiro for fruitful discussions, and also acknowledges support from CNPq, FAPEMIG and

FAPES.

## References

- [1] S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150; see also M. Drees, R.M. Godbole and P. Roy, *Theory and Phenomenology of SPARTICLES: An account of four-dimensional  $N = 1$  Supersymmetry in High Energy Physics*, (World Scientific, 2004).
- [2] A.S. Belyaev and I.L. Shapiro, Phys. Lett. **425B** (1998) 246; Nucl. Phys. **B543** (1999) 20.
- [3] G. de Berredo-Peixoto, J.A. Helayel-Neto and I. L. Shapiro, JHEP **02** (2000) 003.
- [4] I.L. Shapiro, Phys. Repts. 357 (2002) 113.
- [5] E.C.G. Stückelberg, Helv. Phys. Acta. **30** (1957) 209.
- [6] I.L. Buchbinder, G. de Berredo-Peixoto and I.L. Shapiro, *Phys. Lett. B* **649**, 454-462 (2007); G. de Berredo-Peixoto and I.L. Shapiro, *J. Phys A: Math. Theor.* **41**, 164044 (2008).
- [7] F.W. Hehl, Gen. Relat. Grav. **4** (1973) 333; **5** (1974) 491;  
F.W. Hehl, P. Heide, G.D. Kerlick and J.M. Nester, Rev. Mod. Phys. **48** (1976) 393.
- [8] A.O. Barvinsky and G.A. Vilkovisky, Phys. Rep. **119** (1985) 1.